**Exercise: Asymptotic Notation Proof**

1. 3n2 + 10nlog2n = O(nlog2n)  
   here,  
    f(n)=3n2 + 10nlog2n , g(n)=nlog2n  
   We know,

f(n) **≤** cg(n)hence,  
 3n2 + 10nlog2n**≤** cnlog2n 3n + 10log2n**≤** clog2n [Divided by n]  
  
**Here we can see** f(n) is asymptotically faster growing than g(n) so f(n) is not upper bounded by O(g(n))

1. 3n2 + 10nlog2n = Ω(n2)

here,  
 f(n)=3n2 + 10nlog2n , g(n)=n2  
We know,

cg(n) **≤** f(n)hence,  
 cn2 **≤** 3n2 + 10nlog2n c**≤** 3 + 10log2n/n [Divided by n2]

c=3, n0=1

**Here we can see that** since f(n) is asymptotically faster growing than g(n) hence f(n) is lower bounded by Ω(n2).

1. 3n2 + 10nlog2n = Θ(n2)

here,  
 f(n)=3n2 + 10nlog2n , g(n)=n2  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1n2 **≤** 3n2 + 10nlog2n **≤** c2n2 c1**≤** 3 +10log2n/n **≤** c2 [Divided by n]

c1= 3, c2 = 8 and n0 = 1

**Here we can see that** f(n) is both asymptotically faster growing for c1 and asymptotically slower growing for c2 so f(n) is tight bounded by Θ(n2).

1. nlog2n + n/2 = O(n)

here,  
 f(n)= nlog2n + n/2 , g(n)=n

We know,

f(n) **≤** cg(n)hence,  
 log2n + n/2**≤** cn

log2n + 1/2**≤** c [Divided by n]

**Here we can see** f(n) is asymptotically faster growing than g(n) so it is not upper bounded by n.

1. 10√n + log2n = O(n)

here,  
 f(n)=10√n + log2n , g(n)=n

We know,

f(n) **≤** cg(n)hence,  
 10√n + log2n**≤** cn

10/√n + log2n/n**≤** c√n [Divided by n]

c = 10, n0=1

**Here we can see** n is asymptotically faster growing than f(n), so f(n) is upper bounded by g(n).

1. √n + log2n = O(log2n)

here,  
 f(n)=√n + log2n , g(n)=log2n

We know,

f(n) **≤** cg(n)hence,  
 √n + log2n**≤** clog2n

√n/log2n + 1**≤** c [Divided by log2n]  
   
**Here we can see** that f(n) is asymptotically faster growing than g(n) so, f(n) is not upper bounded by g(n).

1. √n + log2n = Θ(log2n)

here,  
 f(n)=√n + log2n , g(n)=log2n  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1log2n **≤** √n + log2n **≤** c2log2n  
 c1 **≤** √n/log2n + 1 **≤** c2 [Divided by log2n]

**Here we can see** that although we can find c1 and n0 for the inequality however we can’t find a c2 that stands for the inequality so, f(n) is not tight bounded by g(n).

1. √n + log2n = Θ(n)

here,  
 f(n)=√n + log2n , g(n)= n  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1n **≤** √n + log2n **≤** c2n  
 c1 **≤** 1/√n + log2n/n**≤** c2 [Divided by n]  
  
**Here we can see** that f(n) can not be asymptotically lower bounded by a constant multiple of g(n), So, f(n) is not tight bounded by g(n).

1. 2n + log2n = Θ(√n)

here,  
 f(n)=2n + log2n , g(n)= √n  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1√n **≤** 2n + log2n **≤** c2√n

c1 **≤** 2√n + log2n/√n **≤** c2 [Divided by √n]

**Here we can see** that f(n) can be asymptotically lower bounded by a constant multiple c1 however it can not be upper bounded by any constant multiple of g(n) So, f(n) can not be tight bounded by g(n).

1. 1/2n2 - 3n = Θ(n2)

here,  
 f(n)=1/2n2 - 3n , g(n)= n2  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1n2 **≤** 1/2n2 - 3n **≤** c2n2

c1 **≤** 1/2 - 3/n **≤** c2

c1= 1/14, c2 = 1/2 and n0 = 7

**Here we can see** that f(n) is both upper and lower bounded by constant multiples of g(n), So, f(n) is tight bounded by g(n).

1. 6n3 = Θ(n2)

here,  
 f(n)=6n3 , g(n)= n2  
We know,

c1g(n) ≤ f(n) ≤ c2g(n)hence,  
 c1n2 **≤** 6n3 **≤** c2n2  
 c1 **≤** 6n **≤** c2, [Divided by n2]

**Here we can see** that f(n) can be asymptotically lower bounded by a constant multiple c1 however it can not be upper bounded by any constant multiple of g(n) So, f(n) can not be tight bounded by g(n).

1. √n + log2n = Ω(1)

here,  
 f(n)=√n + log2n , g(n)= 1  
We know,

cg(n) ≤ f(n) hence,  
 c **≤** √n + log2n

c=1 and n0=1

**Here we can see** that f(n) is asymptotically faster growing than g(n) , so it is lower bounded by g(n).

1. √n + log2n = Ω(log2n)

here,  
 f(n)=√n + log2n , g(n)=log2n  
We know,

cg(n) ≤ f(n) hence,  
 clog2n **≤** √n + log2n ,

c **≤** √n/log2n + 1 [Divided by log2n]

c=1 and n0=1

**Here we can see** that f(n) is asymptotically faster growing than g(n) , so it is lower bounded by g(n).

1. √n + log2n = Ω(n)

here,  
 f(n)=√n + log2n , g(n)=n  
We know,

cg(n) ≤ f(n) hence,  
 cn **≤** √n + log2n

c **≤** 1/√n + log2n/n [Divided by n]

**Here we can see** that f(n) is slower growing than g(n), so it can not be lower bounded by g(n).